Waves - II

V Vijaykiran
M.Sc., B.Ed., NET
SIGMA Physics Resource Centre
http://www.sigmaprc.in

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Stationary waves (theory)

Stationary waves in open pipe

Stationary waves (strings)

Stationary waves in a closed pipe

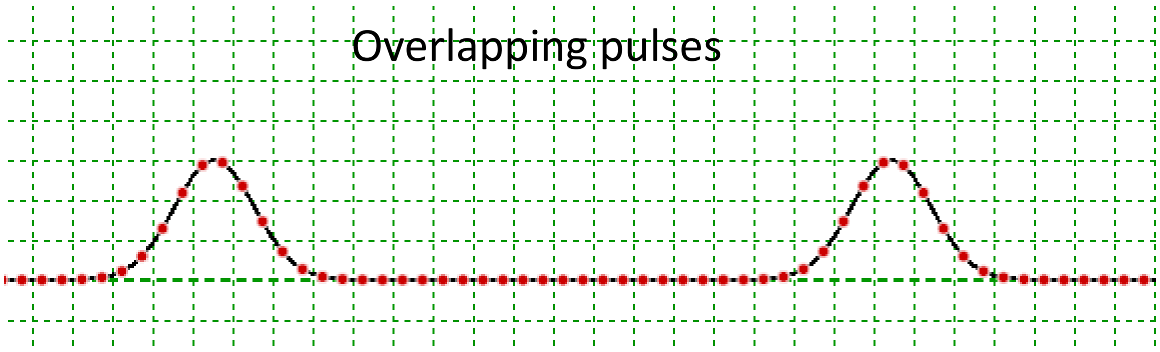
Laws of transverse waves in a string

Resonance column experiment

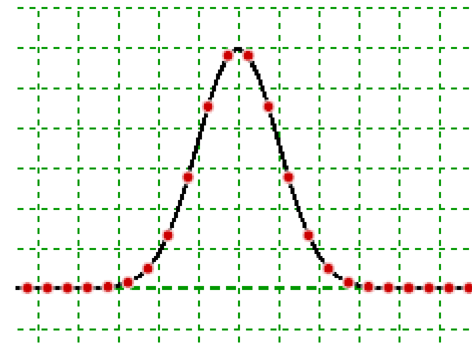
Principle of superposition

- When two or more pulses overlap each other then the resultant displacement is the algebraic sum of the individual displacement due to each pulse.

$$y_n = y_1(x,t) + y_2(x,t) + \ldots$$

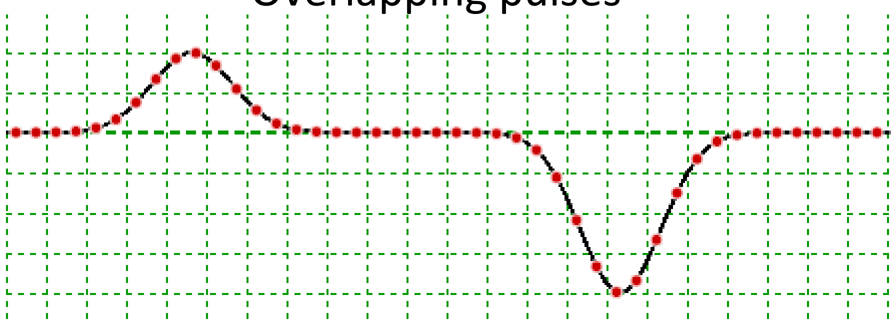


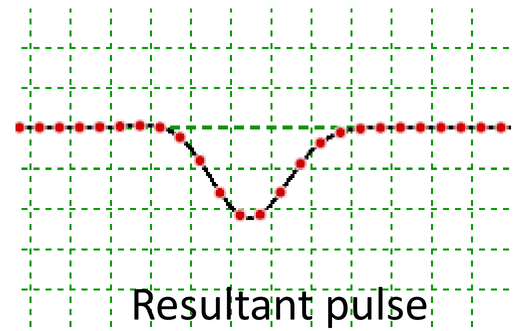
Overlapping pulses



Resultant pulse

Overlapping pulses





Resultant pulse

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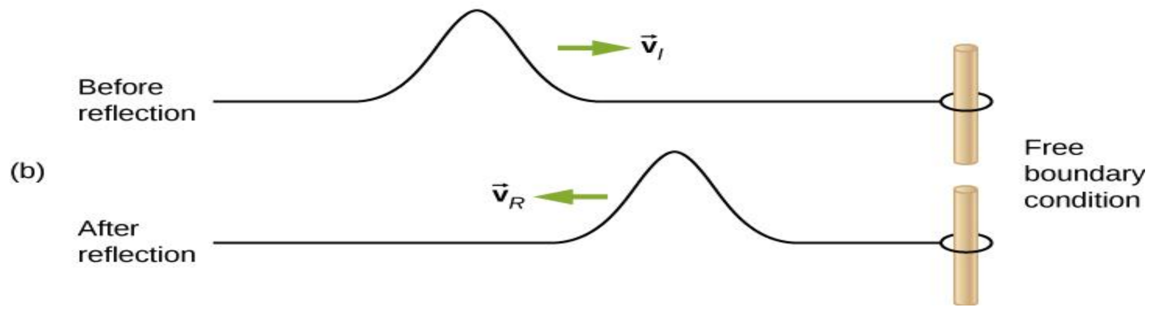
Reflection of waves

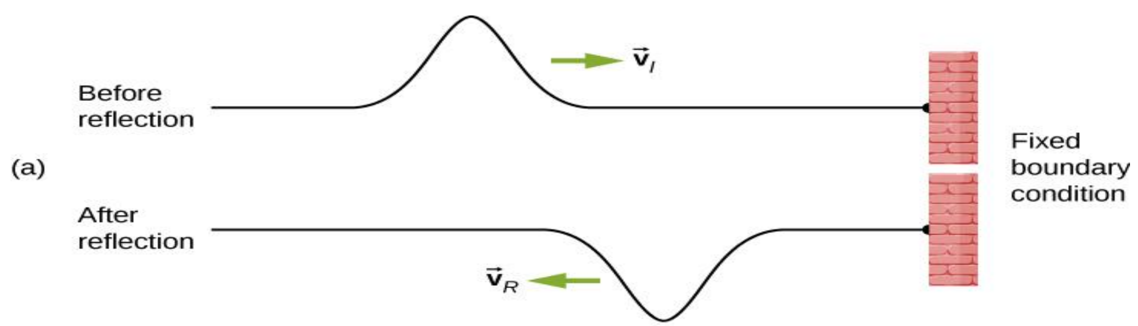
- A wave reflected at an open (or free) end does not undergo any phase change.

$$y_{\text{i}} = A\sin(\omega t \pm kx) \qquad y_{\text{r}} = A\sin(\omega t \mp kx)$$



- A wave reflected at closed end (or a fixed end) undergoes a phase change of π.

$$y_{\text{i}} = A\sin(\omega t \pm kx) \qquad y_{\text{r}} = -A\sin(\omega t \mp kx)$$



Formation of stationary waves

A stationary wave is formed when two waves of same amplitude and wavelength, propagating in opposite directions, superimpose each other.

A wave propagating along negative x-direction may be given by the relation

$$y_1 = A\sin(\omega t + kx)$$

Upon reflection at a fixed end, the reflected wave is given by the relation

$$y_2 = -A\sin(\omega t - kx)$$

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Using principle of superposition we get

$$y = y_1 + y_2$$

$$y = 2A\sin\left(\frac{2kx}{2}\right)\cos\left(\frac{2\omega t}{2}\right)$$

$$y = 2A\sin(kx)\cos(\omega t)$$

- Amplitude depends on position of a particle
- Depending on the superimposing waves, the stationary wave equation may contain other combinations of sine or cosine functions

Nodes and antinodes

Displacement of a particle in a stationary wave is given by the relation

$$y = 2A\sin(kx)\cos(\omega t)$$

Amplitude of a particle is determined by is location (x) $A_{stat} = 2A\sin(kx)$

Nodes

These are the points where amplitude of particles is zero.

$$\sin(kx) = 0$$

$$kx = 0, \pi, 2\pi, 3\pi \ldots m\pi$$

$$\frac{2\pi}{\lambda}x = m\pi$$

$$x = m\frac{\lambda}{2}$$

Antinodes

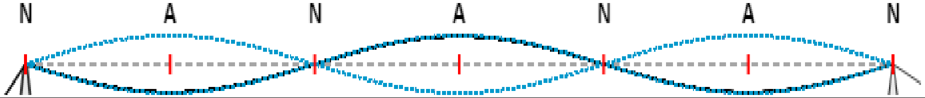
These are the points where amplitude of particles is the maximum ($2A$).

$$\sin(kx) = \pm 1$$

$$kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots \frac{(2m+1)\pi}{2}$$

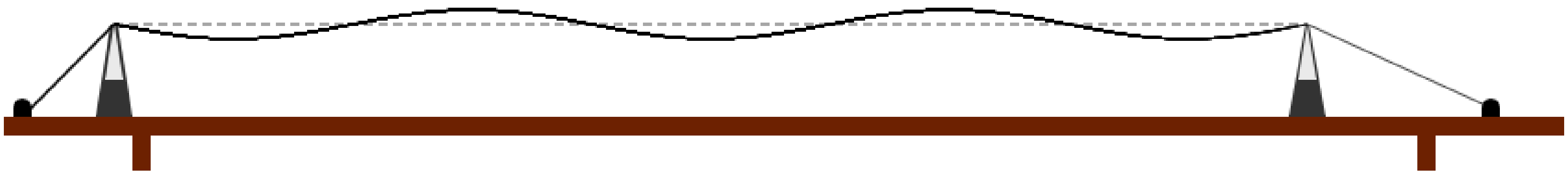
$$\frac{2\pi}{\lambda}x = \frac{(2m+1)\pi}{2}$$

$$x = \frac{(2m+1)\lambda}{4}$$



Stationary waves in a stretched string

Consider a string of length L and linear density μ (mass per unit length) under tension T. A transverse wave generated in the string propagates along it and upon reflection at the fixed end, undergoes a phase change of π. A stationary wave is formed when the incident and the reflected waves superimpose each other.



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A wave propagating along negative x-direction may be given by the relation

$$y_1 = A\sin(\omega t + kx)$$

The reflected wave is given by

$$y_2 = -A\sin(\omega t - kx)$$

Using principle of superposition, the resultant wave is given by

$$y = 2A\sin(kx)\cos(\omega t)$$

Comparing general equation of velocity of a wave to the velocity of transverse wave in a stretched string we get

$$n\lambda = \sqrt{\frac{T}{\mu}} \Rightarrow \boxed{n = \frac{1}{\lambda}\sqrt{\frac{T}{\mu}}}$$

Frequency of vibration can be calculated using the above equation and wavelength in each mode of vibration.

Laws of transverse vibrations in a stretched string

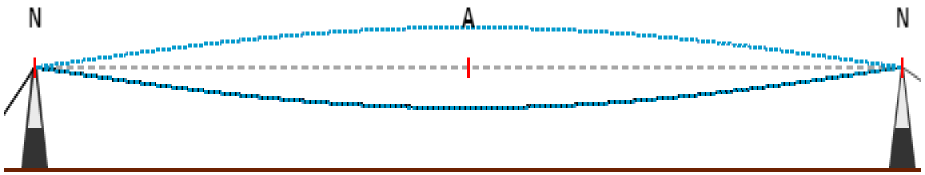
$$n_{\circ} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

1. The fundamental frequency of a stretched string is inversely proportional to the length of vibrating segment, when tension in the string and its linear density are held constant.

2. The fundamental frequency of a stretched string is directly proportional to the square root of tension in the string, when length of vibrating segment and its linear density are held constant.

3. The fundamental frequency of a stretched string is inversely proportional to the square root of linear density of the string, when length of vibrating segment and tension are held constant.

1^{st} mode of vibration

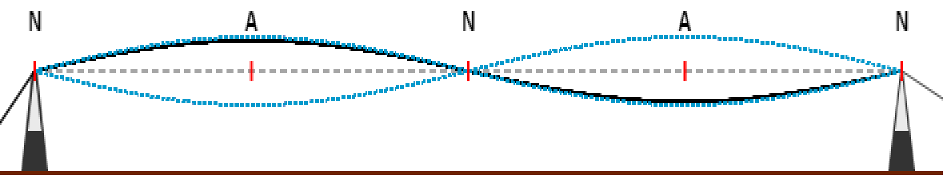


$$L = \frac{\lambda}{2} \qquad \lambda = \frac{2L}{1}$$

$$n_o = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

Fundamental frequency or 1^{st} harmonic

2^{nd} mode of vibration



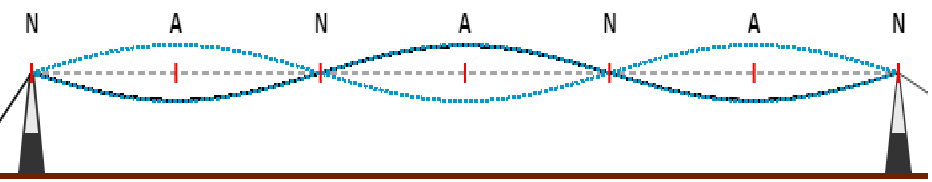
$$L = 2\frac{\lambda}{2} \quad \Rightarrow \lambda = 2\frac{L}{2}$$

$$n = \frac{2}{2L}\sqrt{\frac{T}{\mu}}$$

2^{nd} harmonic

$$n = 2n_o$$

3^{rd} mode of vibration



$$L = 3\frac{\lambda}{2} \quad \Rightarrow \lambda = \frac{2L}{3}$$

$$n = \frac{3}{2L}\sqrt{\frac{T}{\mu}}$$

3^{rd} harmonic

$$n = 3n_o$$

- In a stretched string both odd & even harmonics of fundamental frequency are produced.

For the stationary waves formed in a stretched string

- Fundamental frequency of a stretched string is given by $n_o = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$
- Both odd and even harmonics are produced
- Ratio of frequencies generated is $n_1 : n_2 : n_3 : n_4 = 1 : 2 : 3 : 4$
- Frequency of m^{th} harmonic is equal to frequency of $(m - 1)^{th}$ overtone
- Fundamental (and the higher frequencies) depend on length of the string , tension in the string and its linear density.
- Nodes are always formed at the fixed ends
- Distance between two adjacent nodes (or antinodes) is $\lambda/2$
- Distance between a node and the nearest antinode is $\lambda/4$

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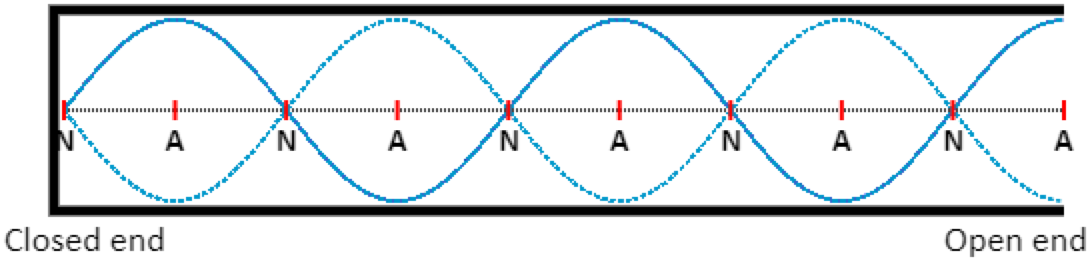
Note :

- Fundamental frequency is the lowest frequency produced in the arrangement
- Harmonics are integral multiples of fundamental frequency
- Any frequency higher than the fundamental frequency is called an overtone.

Stationary waves in a closed organ pipe

An organ pipe is said to be closed when it is open at one end and closed at the other. A stationary wave is formed in a closed pipe when the incident wave superimposes with the wave reflected at the closed end. In any mode of vibration, an antinode is always formed at the free end and a node is always formed at the closed end.



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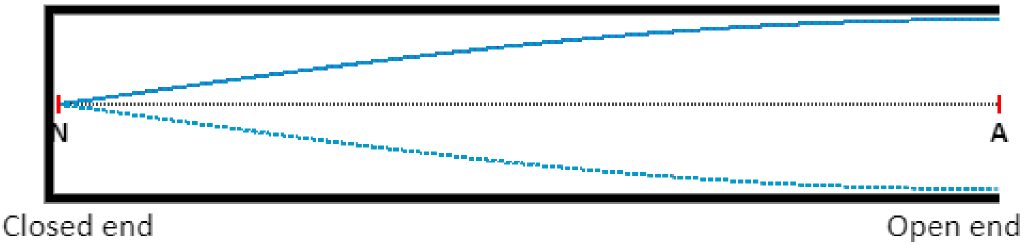
Relation between velocity of wave (v) , wavelength (λ) an the frequency (n) is

$$v = n\lambda \quad \Rightarrow \quad n = \frac{v}{\lambda}$$

Frequencies of the harmonics produced by a closed pipe can be determined by obtaining wavelength of each mode of vibration in terms of length (L) of the pipe.

1st mode of vibration

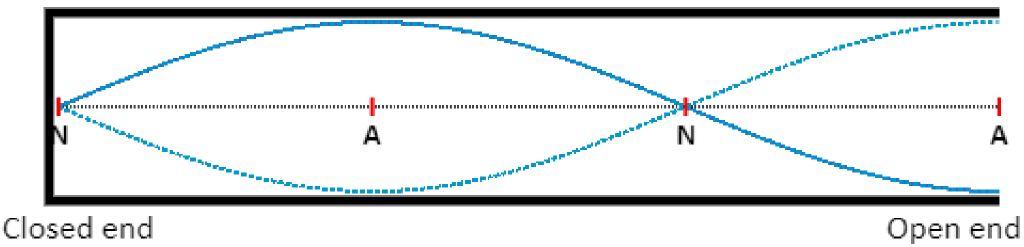


$$L = \frac{\lambda}{4} \Rightarrow \lambda = \frac{4L}{1}$$

$$n_o = \frac{v}{4L}$$

Fundamental frequency or 1st harmonic

2nd mode of vibration



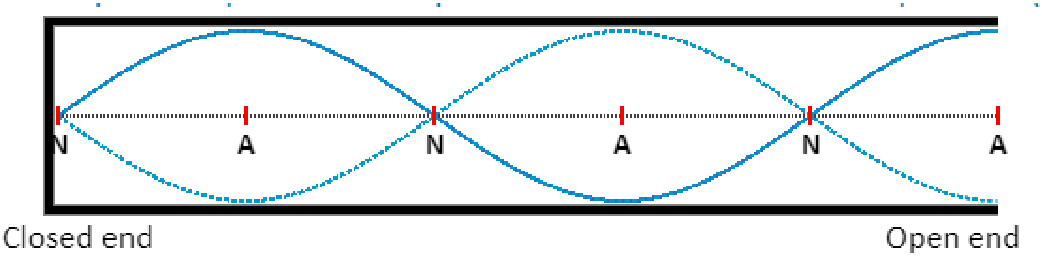
$$L = 3\frac{\lambda}{4} \Rightarrow \lambda = \frac{4L}{3}$$

$$n = \frac{3v}{4L}$$

3rd harmonic

$$n = 3n_o$$

3rd mode of vibration



$$L = 5\frac{\lambda}{4} \Rightarrow \lambda = \frac{4L}{5}$$

$$n = \frac{5v}{4L}$$

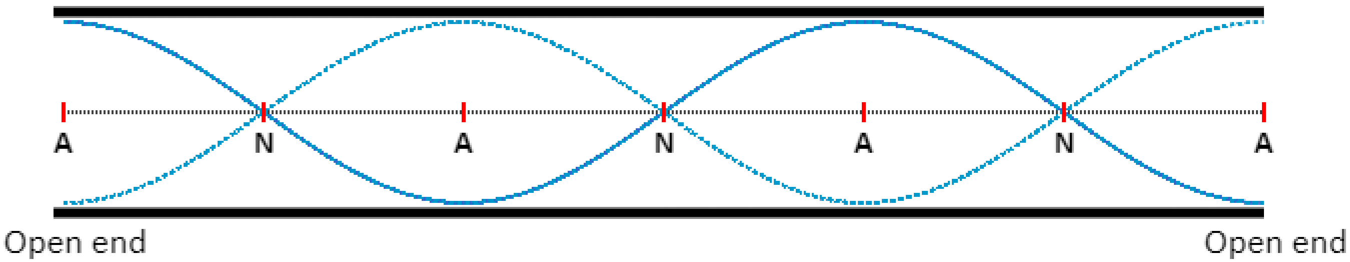
5th harmonic

$$n = 5n_o$$

- In a closed pipe only odd harmonics of fundamental frequency are produced.

Stationary waves in an open organ pipe

An organ pipe is said to be open when it is open at both the ends. A stationary wave is formed in an open pipe when the incident wave superimposes with the wave reflected at the open end. In any mode of vibration antinodes are always formed at both the free ends of the pipe.



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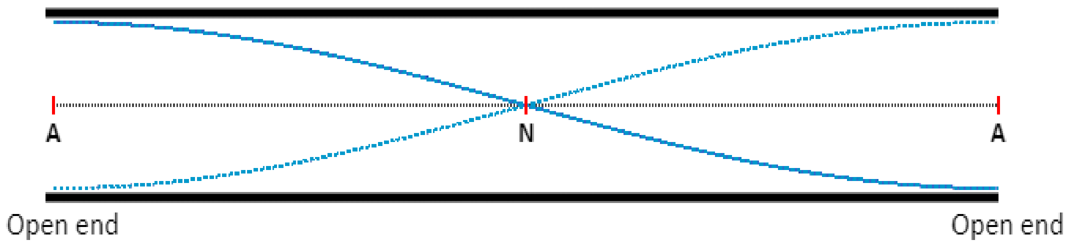
Relation between velocity of wave (v) , wavelength (λ) an the frequency (n) is

$$v = n\lambda \quad \Rightarrow \quad n = \frac{v}{\lambda}$$

Frequencies of the harmonics produced by a closed pipe can be determined by obtaining wavelength of each mode of vibration in terms of length (L) of the pipe.

1st mode of vibration

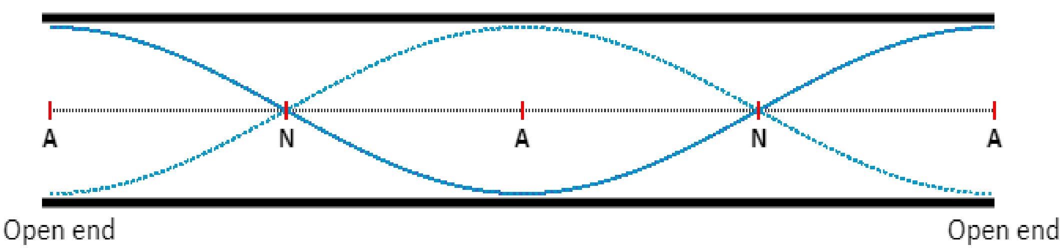


$$L = \frac{\lambda}{2} \quad \Rightarrow \lambda = \frac{2L}{1}$$

$$n_o = \frac{v}{2L}$$

Fundamental frequency or 1st harmonic

2nd mode of vibration



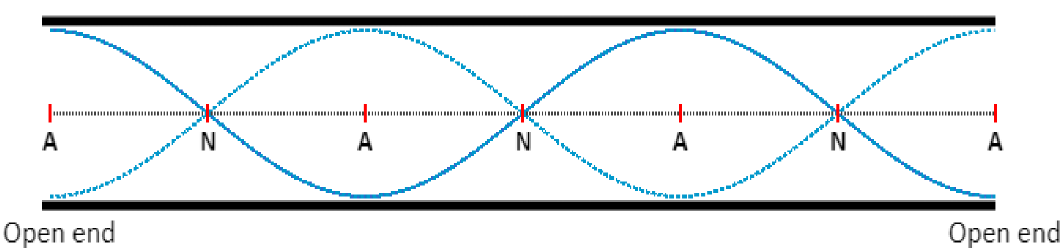
$$L = 2\frac{\lambda}{2} \quad \Rightarrow \lambda = \frac{2L}{2}$$

$$n = 2\frac{v}{2L}$$

2nd harmonic

$$n = 2n_o$$

3rd mode of vibration



$$L = 3\frac{\lambda}{2} \quad \Rightarrow \lambda = \frac{2L}{3}$$

$$n = 3\frac{v}{2L}$$

3rd harmonic

$$n = 3n_o$$

- In an open pipe both odd and even harmonics of fundamental frequency are produced.

A comparative summary of stretched string, open pipe and closed pipe

Stretched string	Open pipe	Closed pipe
Nodes formed at both the fixed ends	Antinodes formed at both the open ends	Nodes at closed end and antinodes at open end
Fundamental frequency $n_o = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$	Fundamental frequency $n_o = \frac{v}{2L}$	Fundamental frequency $n_o = \frac{v}{4L}$
Higher frequencies $2n_o, 3n_o, 4n_o \ldots$	Higher frequencies $2n_o, 3n_o, 4n_o \ldots$	Higher frequencies $3n_o, 5n_o, 7n_o \ldots$
Odd and even harmonics	Odd and even harmonics	Only odd harmonics
$m^{\text{th}}\text{OT} = (m+1)^{\text{th}}\ \text{H}$	$m^{\text{th}}\text{OT} = (m+1)^{\text{th}}\ \text{H}$	$m^{\text{th}}\text{OT} = (2m+1)^{\text{th}}\ \text{H}$

Natural oscillations or free oscillations : Oscillations or vibrations of a body when subjected to an impulsive force.

Example : Striking a drum, plucking a string etc.

Natural frequency: Frequency of natural vibrations (or oscillations) of a body.

Forced oscillations : Oscillations or vibrations of a body when subjected to an external periodic force.
Example : waving flag,

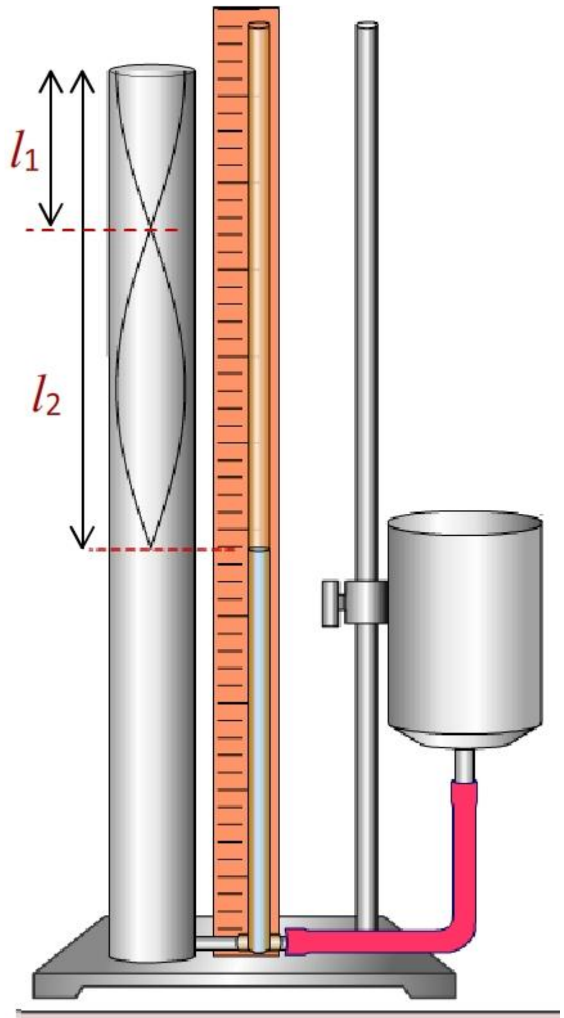
Resonance: It is a special case of forced vibrations in which frequency of external periodic force is equal to the natural frequency of the body resulting in vibrations of increased amplitude.

Examples :

Resonance air column apparatus

<u>Principle</u> : Resonance of tuning fork with stationary waves formed in the air column of a closed wipe

- Water level in the pipe is varied using the reservoir.
- At each level the excited tuning fork of frequency n is placed near the open end of the pipe.
- Formation of stationary waves at a particular level is confirmed by observing a booming sound due to resonance.
- Length of air column when the first resonance is heard is called first resonating length (l_1).
- Length of air column (after lowering the water level further) when the second resonance is heard is called second resonating length (l_2) .
- Velocity of sound (at room temperature) is obtained using the relation $v = 2n(l_2 - l_1)$



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Resonance air column apparatus

Antinode of stationary waves formed in the closed pipe lie slightly beyond the closed end. This is taken into account using end correction (e)

At the first resonating length

$$\frac{\lambda}{4} = l_1 + e$$

At the second resonating length

$$3\frac{\lambda}{4} = l_2 + e$$

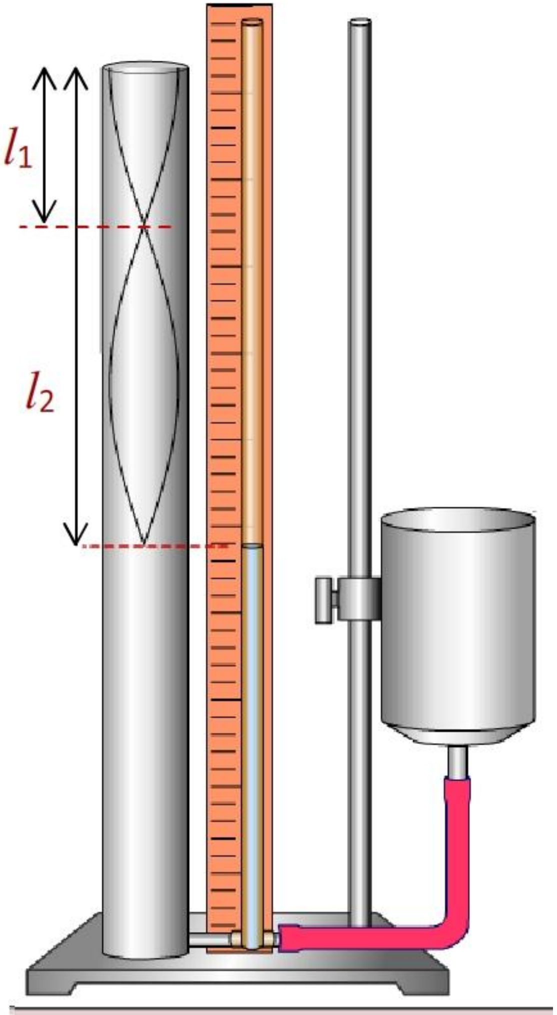
Subtracting equation (i) from equation (ii)

$$\frac{\lambda}{2} = l_2 - l_1$$

$$\Rightarrow \lambda = 2(l_2 - l_1)$$

Using $v = n\lambda$ we get

$$\boxed{v = 2n (l_2 - l_1)}$$



Velocity of sound at 0°C

Velocity of sound in air is given by

$$v = \sqrt{\frac{\gamma P}{d}}$$

Using the ideal gas equation

$$v = \sqrt{\frac{\gamma nRT}{Vd}}$$

$$v = \sqrt{\gamma cT}$$

$$v \propto \sqrt{T}$$

Velocity of sound at 0°C and at any other temperature may be compared as

$$\frac{v}{v_o} = \sqrt{\frac{T}{273}}$$

Representing the temperature in centigrade scale as *t* we get

$$\frac{v}{v_o} = \sqrt{\frac{273+t}{273}}$$

$$\frac{v}{v_o} = \sqrt{1+\frac{t}{273}}$$

$$\frac{v}{v_o} = \left(1+\frac{t}{273}\right)^{1/2}$$

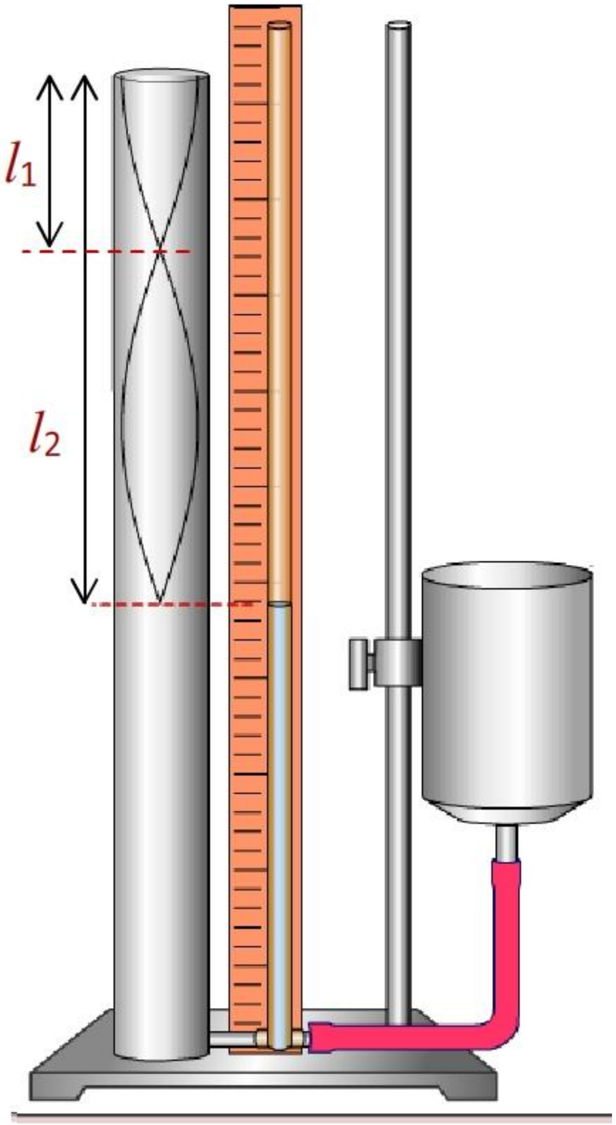
$$v = v_o\left(1+\frac{t}{546}\right)$$

$$\boxed{v_o = \frac{v}{\left(1+\frac{t}{546}\right)}}$$

Resonance air column apparatus

Sources of errors in measurement

- Random error associated with determination of resonating lengths
  This is minimized by taking the average of a number of trials for reach resonating length
- Systematic error in measurement of resonating lengths
  This is minimized by checking alignment of scale and the upper end of pipe
- Systematic error due to antinode formed beyond the open end of pipe
  This is minimized by making the end correction



sigmaprc@gmail.com

sigmaprc.in